Notes on the reality of the quantum state

Shan Gao*

January 6, 2014

Abstract

Based on an analysis of protective measurements, we show that the quantum state represents the physical state of a single quantum system. This result is more definite than the PBR theorem [Pusey, Barrett, and Rudolph, Nature Phys. 8, 475 (2012)].

The physical meaning of the quantum state is an important interpretative problem of quantum mechanics. A long-standing question is whether a pure state relates only to an ensemble of identically prepared systems or directly to the state of a single system. Recently, Pusey, Barrett and Rudolph (PBR) demonstrated that under an independence assumption, the quantum state is a representation of the physical state of a single quantum system [1]. This poses a further interesting question, namely whether ψ-ontology can be argued without resorting to nontrivial assumptions such as the independence assumption (cf. Ref. [2-4]). In this Letter, we will show that protective measurements [5,6] already provide such an argument.

The meaning of the quantum state is usually analyzed in the context of conventional impulsive measurements of an ensemble of identically prepared systems. However, it has been known that the quantum state of a single prepared system can be protectively measured [5-9]. During a protective measurement, the measured system is protected by an appropriate procedure (e.g. via the quantum Zeno effect) so that its quantum state neither changes nor becomes entangled with the quantum state of the measuring device appreciably. In this way, such protective measurements can measure the expectation values of observables from a single quantum system, even if the system is initially not in an eigenstate of the measured observable, and in particular, the quantum state of the system can also be measured as expectation values of certain observables. It is expected that protective measurements will be realized in the near future with the rapid development of quantum technologies.

*Institute for the History of Natural Sciences, Chinese Academy of Sciences, Beijing 100190, P. R. China. E-mail: gaoshan@ihns.ac.cn.

It can be argued that protective measurements provide a strong support for ψ-ontology, according to which the quantum state represents the physical state of a single quantum system. Several authors, including the inventors of protective measurements, have given such arguments [5,6,10,11]. However, these arguments have been neglected by most researchers including Pusey, Barrett and Rudolph [1]. Here we will present a clearer argument for ψ-ontology in terms of protective measurements. According to quantum mechanics, we can prepare a single measured system whose quantum state is $\psi(t)$ at a given instant t. The question is whether the quantum state relates directly to the physical state of the system or merely to an ensemble of identically prepared systems (which is also called state of knowledge). This question can hardly be answered by analyzing a non-protective impulsive measurement of the system (see, e.g. Ref. [1-4]), by which one obtains one of the eigenvalues of the measured observable, and the expectation value of the observable can only be obtained by calculating the statistical average of the eigenvalues for an ensemble of identically prepared systems. Now, by a protective measurement on the measured system, we can directly obtain the expectation value of the measured observable. Moreover, by a series of protective measurements of certain observables on this system, we can also obtain the value of $\psi(t)$. Since we can measure the quantum state *only* from a single prepared system by protective measurements, the quantum state cannot relate only to an ensemble of identically prepared systems, but must directly represent the physical state of a single system.

That the quantum state of a single prepared system can be measured by protective measurements can be illustrated with a specific example [5]. Consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. In this case, the measured system itself supplies the protection of the state due to energy conservation and no artificial protection is needed. We take the measured observable A_n to be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A_n = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{1}$$

An adiabatic measurement of A_n then yields

$$\langle A_n \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 dv, \tag{2}$$

which is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Similarly, we can adiabatically measure another observable $B_n = \frac{\hbar}{2mi}(A_n \nabla + \nabla A_n)$. The measurement yields

$$\langle B_n \rangle = \frac{1}{v_n} \int_{V_n} \frac{\hbar}{2mi}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) dv = \frac{1}{v_n} \int_{V_n} j(x) dv. \tag{3}$$

This is the average value of the flux density $j(x)$ in the region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ and $j(x)$ everywhere in space. Since the wave function $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$ (except for an overall phase factor), the above protective measurements can measure the wave function of the measured system.

We can also give a PBR-like argument for ψ-ontology in terms of protective measurements (cf. Ref. [1]). For two (known) nonorthogonal states of a quantum system, the results of the protective measurements of them are different. If there exists a non-zero probability that these two nonorthogonal states correspond to the same physical state λ, then when assuming λ determines the probability of measurement results as the PBR theorem assumes, the results for the two nonorthogonal states will be the same with the non-zero probability. This leads to a contradiction. This argument, like the above one, only considers a single quantum system, and thus avoids the independence assumption used by the PBR theorem.

There are two possible objections to the above conclusion that protective measurements support the reality of the quantum state. The first is based on the requirement that an unknown ontic state can be measured. It claims that since an unknown quantum state cannot be protectively measured, protective measurements do not have implications for the ontological status of the quantum state. However, this requirement is no doubt too strong. If it were true, then no argument for the reality of the quantum state including the PBR theorem could exist, because it is a well-known result of quantum mechanics that an unknown quantum state cannot be measured. On the other hand, it is also worth noting that protective measurements alone cannot imply the reality of the quantum state. In both the PBR theorem and the above arguments, a realist view on the theory-reality relation is implicitly assumed, which means that the theoretical terms expressed in the language of mathematics connect to the entities existing in the physical world. According to this assumption, the quantum state in quantum mechanics relates either to the state of an ensemble of identically prepared systems or to the state of a single system. The question is to determine which interpretation is true. Here protective measurements can help answer this question. Since we can measure the quantum state from a single system by protective measurements, the quantum state can be regarded as a representation of the physical state of a single system.

The second objection concerns realistic protective measurements. A realistic protective measurement can never be performed on a single quantum system with absolute certainty. For example, for a realistic protective measurement of an observable A in a non-degenerate energy eigenstate whose measurement interval T is finite, there is always a tiny probability proportional to $1/T^2$ to obtain a different result $\langle A \rangle_\perp$, where $\perp$ refers to a normalized state in the subspace normal to the measured state as picked out by the first order perturbation theory [12,13]. It thus claims that this precludes an ontological status for the quantum state. However, this objection is not valid either. On the one hand, the probability of obtaining a different result can be made arbitrarily small in principle when T approaches infinity. Our above arguments are based only on the existence of this limit, in which an ideal protective measurement can be performed on a single quantum system with absolute certainty. On the other hand, it can be argued that even realistic protective measurements also support the reality of the quantum state. The key is to realize that when a realistic protective measurement obtains a different result, the measured quantum state is changed to another state, and what the result reflects is this new state, not the original measured state, while when the measurement obtains the right result, namely the expectation value of the measured observable in the measured quantum state, the measured state is not changed (according to standard quantum mechanics), and it is this result that reflects the original measured state. Therefore, although the probability of a realistic protective measurement obtaining a right result is smaller than one, the existence of the result itself has the same efficiency to derive the reality of the quantum state as ideal protective measurements.

Finally, we note that there might also exist other components of the underlying physical state, which are not measureable by protective measurements and not described by the quantum state, e.g. the positions of the Bohmian particles in the de Broglie-Bohm theory. In this case, the quantum state is still uniquely determined by the underlying physical state, though it is not a complete representation of the physical state. As a result, the epistemic interpretation of the quantum state will be ruled out. Certainly, the quantum state also plays an epistemic role by giving the probability distribution of the results of measurements according to the Born rule. However, this role will be secondary and determined by the complete quantum dynamics that describes the measurement process, e.g. the collapse dynamics in dynamical collapse theories.

References

[1] M. F. Pusey, J. Barrett, and T. Rudolph. On the reality of the quantum state. Nature Phys. 8, 475-478. (2012).

[2] P. G. Lewis, D. Jennings, J. Barrett, and T. Rudolph. Distinct quantum states can be compatible with a single state of reality. Phys. Rev. Lett. 109, 150404 (2012).

[3] M. S. Leifer and O. J. E. Maroney. Maximally epistemic interpretations of the quantum state and contextuality. Phys. Rev. Lett., 110, 120401 (2013).

[4] M. K. Patra, S. Pironio, and S. Massar. No-go theorems for ψ-epistemic models based on a continuity assumption. Phys. Rev. Lett., 111, 090402 (2013).

[5] Y. Aharonov and L. Vaidman. Measurement of the Schrödinger wave of a single particle, Phys. Lett. A 178, 38 (1993).

[6] Y. Aharonov, J. Anandan, and L. Vaidman. Meaning of the wave function, Phys. Rev. A 47, 4616 (1993).

[7] Y. Aharonov, J. Anandan, and L. Vaidman. The meaning of protective measurements, Found. Phys. 26, 117 (1996).

[8] L. Vaidman. Protective measurements, in D. Greenberger, K. Hentschel, and F. Weinert (eds.), Compendium of Quantum Physics: Concepts, Experiments, History and Philosophy. Springer-Verlag, Berlin. pp.505-507 (2009).

[9] S. Gao (eds.). Protective Measurements and Quantum Reality: Toward a New Understanding of Quantum Mechanics. Cambridge: Cambridge University Press. Forthcoming 2014.

[10] J. Anandan. Protective measurement and quantum reality. Found. Phys. Lett., 6, 503-532 (1993).

[11] M. Dickson. An empirical reply to empiricism: protective measurement opens the door for quantum realism. Philosophy of Science 62, 122 (1995).

[12] N. D. H. Dass and T. Qureshi. Critique of protective measurements. Phys. Rev. A 59, 2590 (1999).

[13] S. Gao. On Uffink's criticism of protective measurements, Studies in History and Philosophy of Modern Physics, 44, 513-518 (2013).